A WAVELET FILTER CRITERION FOR AN *A-PRIORI* EVALUATION OF WAVELET CODING AND DENOISING PERFORMANCES

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ABSTRACT

Wavelet transforms are used in number of important signal and image processing tasks including image coding. The choice of the filter bank is very important and is directly linked to the efficiency of the compression. An objective criterion to guide the choice of the wavelet filters is proposed. It is composed of two indexes. The first one is a frequency index computed from the aliasing of the filters. The second is a spatial index computed from the spread of the coefficients in spatial domain. The quality of a filter is a trade-off between frequency and spatial quality. From these indexes a filter set can be represented by a point in a plan. The abscissa is given by the frequency index and the ordinate by the spatial index. The criteria is computed for various filters that are represented in the defined plan. This gives a tool for comparing wavelet filters. In a second time the coding performances of the filters are estimated. The denoising performances are also estimated. The results shows that the two proposed indexes allow a good estimation of the coding and denoising performances of the wavelet filters.

1. INTRODUCTION

Fast Wavelet Transforms (FWT) are used in a number of important signal and image processing tasks including image coding and signal denoising. An image is decomposed into a pyramid of embedded approximation and detail images. At each scale the approximation image contains the approximation and the detail images at the next scale. Image coding is performed by allocating bandwidth according to the information contained in these approximation and detail images. This allocation is then followed by a quantization[1].

The choice of the filter bank is very important and is directly linked to the compression performances. It is still an open problem to choose a filter set for FWT image coding. Some criteria have been proposed such as regularity[12], size of the support of the wavelet and number of vanishing moments. The size of the wavelet support increases with the number of vanishing moments. The wavelet regularity is important to reduce the artifacts. The choice of an optimal wavelet is thus the result of a trade-off between the number of vanishing moments and artifacts[10, chapter XI]. But there is only a partial correlation between filter regularity and reconstructed image quality. Villasenor has proposed a framework to compare filters [15]. The comparison is based on computations from impulse response, sidelobe strength and shift-variance minimization. Their two criteria based on the impulse response and step response need heavy computations. Furthermore no correlation between these criteria and the compression quality is given. The aim of this paper is to propose a unique simple criteria that allows the final compression quality estimation.

The interest of the wavelet transform analysis is the deal between frequency and spatial analysis. When a FWT is used to compute the coefficients, the performances of the transform are the performances of the filter bank. A "good" filter set must be efficient both in the frequency and in the spatial domains. In the spatial domain, the quality of a filter set can be estimated from the support of the coefficients of the impulse response of the filter bank. In the frequency domain, the quality of the filter set can be estimated from the aliasing of the filter bank. Two indexes can thus been deduced from these considerations.

Section 2 presents the definition of the spatial and frequency indexes. Section 3 details the data used to estimate compression and denoising quality. Section 4 links the indexes to the data. A formula is given to estimate the coding quality from the spatial and frequency indexes. Another formula is givne to estimate the denoising quality from the same these indexes. The proposed works are preliminary and some improvements, tests and questions remains. They are presented in the conclusion.

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2. METHODOLOGY

In a FWT, the coefficients of the approximation and detail images at each scale are down-sampled. This algorithm is thus not shift-invariant. This phenomena is due to the aliasing introduced by the down-sampling[14, 4]. Several works have presented translation-invariant representations[11, 9, 13]. These representations are constructed by a recombination of the decimated coefficients. At each scale, the odd and even coefficients are combined. Others methods are based on the choice of scaling functions that produce less sensible analyses[3]. As FWT tasks are not based on these representations, it is important to use classical FWT associated with almost-shift-invariant filter sets. It has previously been emphasized that reducing the aliasing amounts to the same to reduce the shift-variance of the analysis[10, chapter XI][15]. An index based on the aliasing estimation is thus a good idea. The other proposed index is based on the support of filter bank impulse response. These two indexes are easily computed from the filter bank~.

2.1. Frequency and spatial indexes

The aliasing of a filter bank is due to the overlapping of the frequency responses of the low-pass and the high-pass filters. Measuring this overlapping is equivalent to measuring the frequency quality of the FWT. We propose the following formula to compute the frequency index :

$$I_f = \int_0^{\pi} |g(\omega)h(\omega)|\, d\omega. \tag{1}$$

The spatial quality of a filter bank can be assessed from the size of the significant support of the equivalent filter. This equivalent filter takes into account the impulse responses of the low-pass and the high-pass filters. The spatial index is inspired by the variancy formula. The variancy measures data dispersion. The expression of the spatial index is thus :

$$I_s = \sum_{k=0}^{\infty} |h * g[k]|\, k^2. \tag{2}$$

where $*$ denotes the convolution operator.

It must be noted that these indexes can be computed even for orthogonal and biorthogonal filter banks with the same equations.

2.2. Indexes interpretation

The two defined indexes were computed for B-spline orthogonal wavelet filters [8, 2] and for orthogonal compactly supported Daubechies filters [6]. The motivation of signal processing with wavelets is due to the spatio-frequential decomposition. A "good" analysis present a trade-off between

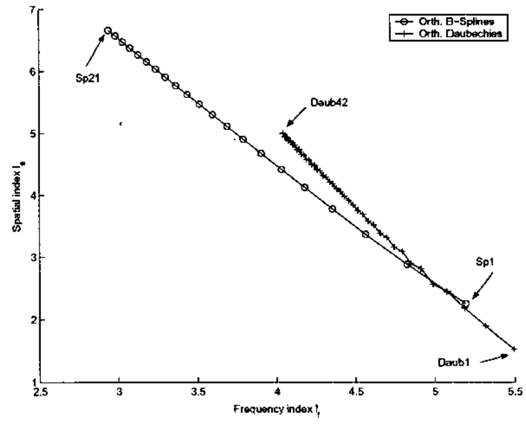

Figure 1: Spatial index with respect to frequency index. The indexes are computed for orthogonal B-spline and orthogonal Daubechies wavelet filters. The axes are in logarithmic scale. These curves show that some filters have a better spatio-frequential trade-off than others as they are closer to the origin.

spatial and frequency analysis. It is thus a good idea to represent a filter bank by a point in the plan defined by the two indexes. Figure 1 shows that when the spatial index increases, the frequency index decreases. This behavior is consistent with the Heisenberg-Gabor uncertainty relation[10, chapter IV]. Two observations can be made :

1. The filters of family form an ordered and monoton set in the plan defined by the two indexes.
2. The more closer to the origin a filter is, the better is the trade-off between spatial and frequency analysis. This shows that it *a-priori* exists "better" filters than others and the measure of a distance with respect to the origin of the plan will allow the assessment of that quality.

2.3. What is a "good" filter ?

We have shown that some filter set show a better trade-off between frequency analysis and spatial analysis than others. But how is this trade-off linked to the properties of the filters ? One of the main question when one want's to use wavelets is : which wavelet will give the best results ? Wavelet transforms are used in a large number of important signal and image processing tasks. Among these tasks, signal denoising and image coding are very important (*e.g.* wavelets are used in JPEG2000 coding). It is easy to compare two coding of the same image by computing their Peak Signal to Noise Ratio (PSNR). We have thus decided for this paper to choose as "good" filters the filters that provides high PSNR in image coding and signal denoising.

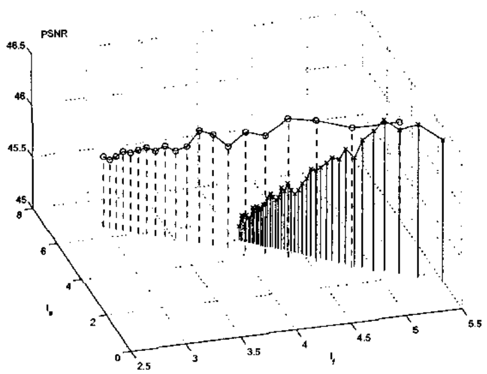

Figure 2: Compression quality of studied filter banks. The mean PSNR is plotted with respect to the spatial and frequency indexes.

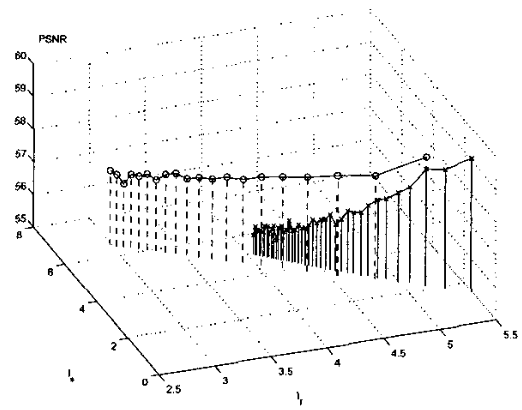

Figure 3: Denoising quality. The mean PSNR is plotted with respect to the indexes.

3. DATA

Two families of filter set were studied : the orthogonal B-spline and the compactly supported orthogonal Daubechies. For each filter set the spatial index I_s, the frequency index I_f and the PSNR between the original image (or signal) and the reconstructed image (or signal) after compression (or denoising) were computed.

For the compression, the coding algorithm is the wavelet transform-based image coder for grayscale images. The transform coder consists of 3 basic steps : 1/a fast wavelet transform (decimated) is performed on the image, 2/the transform coefficients are quantized (discretized) and 3/the quantized coefficients are entropy coded. The coding software was taken from Geoff Davis homepage. This coder is simple but quite effective and is enough to compare filter sets performances. To empirically evaluate the performance of each filter set and to allow comparison with the spatial and frequency indexes, we performed compression on images from the USC-SIPI database[5]. A total of 28 256×256 8-bit test images were used, with the average PSNR computed for a compression of 2:1. The mean PSNR (the PSNR is meaned for the 28 test images) is plotted with respect to I_s and I_f in Fig. 3.

For the denoising, the algorithm was the wavelet shrinkage algorithm[7]. The algorithm shrinks wavelet coefficients with a threshold computed at each scale with the VISUshrink method. To empirically evaluate the performance of each filter set and to allow comparison with the spatial and frequency indexes, we performed denoising on synthetic signals. The synthetic signals and the denoising algorithm was taken from the WaveLab matlab package[16]. For each tested filter bank, 100 noised signals were denoised. The mean PSNR is plotted with respect to I_s and I_f in Fig. 4.

4. RESULTS

Is it possible to estimate the compression PSNR without performing the compression by itself ? In other words, is it possible to estimate the PSNR from I_s and I_f. What is the function that fits $PSNR = f(I_s, I_f)$? For this paper we assume that this function has the following planar form :

$$f(I_s, I_f) = A_0 + A_1 I_f + A_2 I_f \qquad (3)$$

In order to compute the coefficients A_k we performed a linear regression by minimizing $d(PSNR, f(I_s, I_f)) = \sqrt{PSNR^2 + f(I_s, I_f)}$.

For the compression, the obtained coefficients are $A_0 = 64.95$, $A_1 = -2.90$ and$A_2 = -1.62$. Using these coefficients we can estimate the coding quality of a given set of filter with $PSNR_{estimated}(I_s, I_f) = 64.95 - 2.90 I_f - 1.62 I_s$ where I_s and I_f are computed from the filters using eqs. (1) and (4). The next table indicates the quality of the estimation. The errors between the PSNR estimation (obtained from our indexes) and the real value of the PSNR were computed. The greatest error equals 0.2 dB and the mean error equals 0.07dB. This shows that our proposed criterion based on the computation of two indexes is efficient to estimate the wavelet image coding quality.

max error	0.2101dB
$\sqrt{mse}$	0.0677dB

For the denoising, the same model is used. The linear regression allow to obtain the following coefficients : $A_0 = 105.07$, $A_1 = -7.39$ and $A_2 = -3.07$. Using these coefficients we can estimate the denoising quality of a given set of filter with $PSNR_{estimated}(I_s, I_f) = 105.07 - 7.39 I_f - 3.06 I_s$. The next table indicates the quality of the estimation. The errors between the PSNR estimation (obtained from our indexes) and the real value of the PSNR were computed. The greatest error equals 1.12 dB and the mean error equals 0.28dB. This shows that our proposed criterion based on the computation of two indexes is efficient to estimate the wavelet signal denoising quality.

max error	1.12dB
$\sqrt{mse}$	0.28dB

The positive value of A_0 and the negative value of A_1 and A_2 suggest that the maximum value possible for a PSNR in image coding is A_0. This is necessary a limitation of our model. This suggest that the planar hypothesis is limited. Intuitively $I_s = I_f = 0$ would correspond to an infinite PSNR. A better form for the function f would thus be an exponential or logarithmic form.

5. CONCLUSION

We have proposed two indexes to characterize the filter bank of a fast wavelet transform. The first is a frequency index computed from the aliasing of the filters. The second is a spatial index computed from the spread of the coefficients in the spatial domain. A filter bank can be represented by a point in a plan defined by these two indexes. From these indexes it is possible to estimate the quality of an wavelet image coding and signal denoising. We provide a criterion computed from the two indexes that allow the estimation of the PSNR obtained from a images coding and signals denoising. The proposed indexes are efficients for the presented applications but this paper only deals with a work in progress. Some questions and explorations are still opened :

1. The indexes needs to be normalized.
2. The indexes formulaes needs to be connected to the the kernel size in the Heisenberg-Gabor formula. The form of the spatial index is close to the form of the spatial part of the kernel. This suggest that the connection is possible.
3. The curves of Fig.1 suggest a relation between the progression of the spatial index and the frequency index. What is this relation ? This question agree with the second question.
4. More filter sets must be used in order to obtain more accurate results. In particular biorthogonal filter set must be included in the test. This would be easy as the computation of the indexes is the same for orthogonal and biorthogonal filters.

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